Comment

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At the outset I would like to congratulate Ker-Chau Li for developing a new data analytic tool and for providing such a substantial study of it. Regarding the technique itself, an issue that arises quickly is just how robust is it to departures from the fundamental assumption of Section 3? To address this somewhat I attempted to employ the technique on some spatial-temporal global meteorological data. I was interested in studying motion present in this data. Suppose that $Y(x, y, t)$ denotes the measurement made at location (x, y) at time t. Suppose that energy is propagating as a plane wave. The motion may then be represented as

$$f(\alpha x + \beta y + \gamma t)$$

for a function f. This corresponds to movement in direction ϕ, given by $\tan \phi = \beta/\alpha$, with speed $\gamma/\sqrt{\alpha^2 + \beta^2}$. If two such waves are present then one can comtemplate a model

$$Y(x, y, t) = f_1(\alpha_1 X + \beta_1 y + \gamma_1 t) + f_2(\alpha_2 x + \beta_2 y + \gamma_2 t) + noise.$$

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In the circumstance of concern, the measurements were made for (x, y, t) on a lattice. Such data values, (x, y, t), do not satisfy the critical assumption of Section 3. In an attempt to have such a condition obtain I proceeded as follows. Consider a normal distribution $\phi(x)\phi(y)\phi(t)$ centered on the domain of measurements. Obtain realizations of this distribution and let (x_j, y_j, t_j) denote the location on the lattice closest to the jth realization. Now subject the data $(x_j, y_j, t_j, Y(x_j, y_j, t_j))(j = 1, \ldots, n)$ to the analysis of the article. I wondered if Ker-Chau Li's technique would detect motion of weather fronts. I have to report that I was not successful. However the technique certainly did work with corresponding simulated data. (In the simulations, there was a single wave, and f was the cosine function.) There are clearly many things going on in the example, so the failure is not discouraging. The analysis was carried out in S. This package is so widely used now, so one thing I recommend to Ker-Chau Li is that he prepare versions of his programs in S.

I would like to end by mentioning how satisfied Ker-Chau Li's thesis supervisor, Jack Kiefer, would surely have been to see how Ker-Chau Li's work has become such a fine blend of theory and practice.

Comment

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The article by Li proposes a new and very useful approach to dimensionality reduction in multivariate nonparametric regression. The advantage of this approach as compared to others is the exceptional simplicity both of the idea and of the computational tools. We suppose that this would give rise to a wide implementation of sliced inverse regression (SIR).

As with many simple ideas, of course, SIR will also have its pitfalls in "nonsimple" situations. In particular, SIR depends very much on the probability structure of the $\mathbf{x}$ variables described by the following:

For any b in $\mathbf{R}^p$, the conditional expectation $E(b\mathbf{x} \mid \beta_1\mathbf{x}, \ldots, \beta_K\mathbf{x})$ is linear in $\beta_1\mathbf{x}, \ldots, \beta_K\mathbf{x}$; that is, for some constants, $c_0, c_1, \ldots, c_K$,

$$E(b\mathbf{x} \mid \beta_1\mathbf{x}, \ldots, \beta_K\mathbf{x}) = c_0 + c_1\beta_1\mathbf{x} + \cdots + c_K\beta_K\mathbf{x}. \quad (3.1)$$

A nonsimple situation might be where the distribution of $\mathbf{x}$ is a mixture of two normal distributions or has a more complicated nonelliptical structure. In this case, a nonparametric technique based on estimating the multivariate density of $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_p)$ might be reasonable to check the assumption (3.1). We discuss an approach based on this (more complicated) technique later.

There are at least two questions that are important for a practitioner: How to choose the number of principal direc-

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